

DOMAIN WALLS WITH MASSIVE HYPERMULTIPLETS *

MASATO ARAI

*Institute of Physics, AS CR, 182 21, Praha 8, Czech Republic*MUNETO NITTA[†]*Department of Physics, Purdue University, West Lafayette, IN 47907-1396,
USA*

NORISUKE SAKAI

Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, JAPAN

Massive hypermultiplets admit degenerate discrete vacua only if they form non-linear sigma models or have gauge interaction. We discuss BPS domain walls in these theories. This talk is based on the original papers.^{1,2}

1. Introduction

Domain walls play a central role in a subject of recent interest, the brane-world scenario.³ In particular, BPS domain walls are investigated in detail in $D = 4$, $\mathcal{N} = 1$ SUSY theories.⁴ We should consider hypermultiplets to realize $D = 4$, $\mathcal{N} = 1$ SUSY theory on the world-volume. In order to have a potential with discrete degenerate vacua, they must have gauge interaction, or nonlinearity of kinetic term, namely nonlinear sigma models (NLSM). The latter case can be obtained from the former case in the strong gauge coupling limit. A lot of interesting solitons have been discussed in hypermultiplets with⁵ or without⁶ taking this limit.

In this talk, the hyper-Kähler (HK) quotient method⁷ is generalized to the massive models and the BPS domain wall in the simplest case is given. Keeping essential properties of eight supercharges, we discuss a simpler and

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[†]Current address : Tokyo Institute of Technology.

familiar case of $D = 4$, $\mathcal{N} = 2$ SUSY theories in the $\mathcal{N} = 1$ superspace formalism. Brief reviews can be found in reports.⁸ The Harmonic superspace formalism is discussed in the original paper.¹

2. Walls in Chiral Multiplets

We discuss BPS walls in $\mathcal{N} = 1$ SUSY theories with chiral superfields, whose bosonic Lagrangian is $\mathcal{L}_{\text{boson}} = -g_{ij*} \partial_\mu \phi^i \partial^\mu \phi^{*j} - g^{ij*} \partial_i W \partial_{j*} W^*$ with g_{ij*} the Kähler metric. Assuming a domain wall configuration perpendicular to z , its energy density per unit area in the x - y plane is given by

$$\begin{aligned} E &= \int dz (g_{ij*} \partial_z \phi^i \partial_z \phi^{*j} + g^{ij*} \partial_i W \partial_{j*} W^*) \\ &= \int dz |\partial_z \phi^i - e^{i\alpha} g^{ik*} \partial_{k*} W^*|^2 + \int dz (e^{i\alpha} \partial_z \phi^i \partial_i W + \text{c.c.}) \\ &\geq \int dz (\partial_z \phi^i \partial_i W + \text{conj.}) = 2\text{Re}(e^{i\alpha} \Delta W) \end{aligned} \quad (1)$$

with the norm defined by $|V^i|^2 \equiv g_{ij*} V^i V^{*j}$, $\Delta W \equiv W|_{z=\infty} - W|_{z=-\infty}$ and α an arbitrary real constant. Since we obtain the best bound at $e^{-i\alpha} = \Delta W/|\Delta W|$, we derive the BPS bound $E \geq 2|\Delta W|$ saturated by solutions of the BPS equation $\partial_z \phi^i = e^{-i\alpha} g^{ij*} \partial_{j*} W^*$. Since the SUSY transformation on the fermion ψ^i is calculated as $\delta_\epsilon \psi^i = \sqrt{2}(i\sigma^\mu \bar{\epsilon} \partial_\mu \phi^i + \epsilon F^i) = \sqrt{2}(i\sigma^z \bar{\epsilon} e^{-i\alpha} - \epsilon) g^{ij*} \partial_{j*} W^*$, two SUSYs satisfying $ie^{-i\alpha} \sigma^z \bar{\epsilon} = \epsilon$ are preserved.

3. Walls in Hypermultiplets

Let (Φ, Ψ) be $\mathcal{N} = 2$ hypermultiplets with Φ and Ψ being $N \times M$ and $M \times N$ matrix chiral superfields. To obtain nontrivial vacua we need an $U(M)$ gauge symmetry introducing $\mathcal{N} = 2$ vector multiplets (V, Σ) with V an $M \times M$ matrix vector superfield and Σ an $M \times M$ matrix chiral superfield. We work out for the $U(M)$ gauge group in which $U(1)$ part is essential to obtain discrete vacua. We consider the Higgs branch of the theory taking the strong coupling limit $g \rightarrow \infty$ of gauge interactions, which eliminates the kinetic terms for V and Σ . The gauge invariant Lagrangian is given by

$$\begin{aligned} \mathcal{L} &= \int d^4\theta [\text{tr}(\Phi^\dagger \Phi e^V) + \text{tr}(\Psi \Psi^\dagger e^{-V}) - c \text{tr} V] \\ &\quad + \left[\int d^2\theta \{ \text{tr} \{ \Sigma (\Psi \Phi - b \mathbf{1}_M) \} + \sum_{a=1}^{N-1} m_a \text{tr}(\Psi H_a \Phi) \} + \text{c.c.} \right], \end{aligned} \quad (2)$$

with $b \in \mathbf{C}$ and $c \in \mathbf{R}$ constituting a triplet of the Fayet-Iliopoulos parameters, m_a complex mass and H_a Cartan generators of $SU(N)$.^a Eliminating superfields V and Σ using their algebraic equations of motion, we obtain the Lagrangian in terms of independent superfields, in which the Kähler potential is

$$K = c \operatorname{tr} \sqrt{\mathbf{1}_M + \frac{4}{c^2} \Phi^\dagger \Phi \Psi \Psi^\dagger} - c \operatorname{tr} \log \left(\mathbf{1}_M + \sqrt{\mathbf{1}_M + \frac{4}{c^2} \Phi^\dagger \Phi \Psi \Psi^\dagger} \right) + c \operatorname{tr} \log \Phi^\dagger \Phi, \quad (3)$$

with a gauge fixing^b

$$\Phi = \begin{pmatrix} \mathbf{1}_M \\ \varphi \end{pmatrix} Q, \quad \Psi = Q(\mathbf{1}_M, \psi), \quad Q = \sqrt{b}(\mathbf{1}_M + \psi\varphi)^{-\frac{1}{2}}, \quad (4)$$

with φ (ψ) an $(N - M) \times M$ [$M \times (N - M)$] matrix chiral superfield, and the superpotential is

$$W = b \sum_a m_a \operatorname{tr} \left[H_a \begin{pmatrix} \mathbf{1}_M \\ \varphi \end{pmatrix} (\mathbf{1}_M + \psi\varphi)^{-1} (\mathbf{1}_M, \psi) \right]. \quad (5)$$

This is the massive extension of the HK NLSM on the cotangent bundle over the Grassmann manifold, $T^*G_{N,M}$, found by Lindström and Roček.⁷ This model contains ${}_N C_M = N!/M!(N - M)!$ discrete degenerate vacua corresponding to independent gauge fixing conditions (4).¹

The $M = 1$ case of $U(1)$ gauge symmetry reduces to $T^*\mathbf{CP}^{N-1}$ with the superpotential, which admits N parallel domain walls. Moreover if we take $N = 2$ and $M = 1$, the target space $T^*\mathbf{CP}^1$ is the Eguchi-Hanson space with the superpotential $W = b \frac{\mu}{1 + \varphi\psi}$ ($\mu \equiv m_1$). The BPS equation in $\mathcal{N} = 1$ superfields can be solved to give²

$$\varphi = \psi^* = e^{|\mu|(z - z_0)} e^{i\delta}, \quad (6)$$

where z_0 and δ are integral constants. They correspond to zero modes arising from spontaneously broken translational invariance perpendicular to domain wall configuration and $U(1)$ isometry σ_3 in the *internal* space.

^aFlavor symmetry $\Phi \rightarrow \Phi' = g\Phi$, $\Psi \rightarrow \Psi' = \Psi g^{-1}$ with $g \in SU(N)$ in the massless limit $m_a = 0$ is explicitly broken by the mass to its Cartan $U(1)^{N-1}$ generated by H_a .

^bWe discuss the $b \neq 0$ case here. The $b = 0$ case must be discussed independently.¹

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